# 11<sup>th</sup> Annual Johns Hopkins Math Tournament
## Sunday, April 11, 2010
### Pitfalls of Algebra

(1) (**6**) Find the sum
$$1 + 3 + 5 + \cdots + 97 + 99$$

(2) (**7**) Let $a$ be any integer in the set $\{0, 1, \ldots, 9\}$. For how many $a$ is the number $3a512a46a$ divisible by 7 and only 7?

(3) (**8**) Determine the number of consecutive zeroes at the right end of the decimal expansion of
$$\left( \left( \left( 2010^{2010^{2009}} \right)^{2008} \right)^{\cdot^{\cdot^{\cdot}}} \right)^{1}$$
where $2010 = 2 \cdot 3 \cdot 5 \cdot 67$.

(4) (**10**) Find the real number $x$ such that
$$x + 2x^2 + 3x^3 + 4x^4 + \cdots = 30$$

(5) (**11**) Let $a, b, c, d > 0$ be real numbers such that $a + b + c + d = 6$. Find the minimum value of
$$\left( a + \frac{1}{b} \right)^2 + \left( b + \frac{1}{c} \right)^2 + \left( c + \frac{1}{d} \right)^2 + \left( d + \frac{1}{a} \right)^2$$

(6) (**13**) Compute
$$\frac{1}{e \times \pi} + \frac{1}{\pi \times (2\pi - e)} + \frac{1}{(2\pi - e) \times (3\pi - 2e)} + \frac{1}{(3\pi - 2e) \times (4\pi - 3e)} + \cdots$$

(7) (**14**) Find the sum of all integers $0 \le x \le 100$ such that $f(x) = x^2 - 3x + 27$ is divisible by 37.

(8) (**15**) What is the largest integer less than or equal to $(\sqrt{3} + \sqrt{2})^6$?

(9) (**16**) Let $A$ be a set of real numbers such that there always exists $x$, $y$ in $A$ with the following property:
$$0 \le \frac{x - y}{1 + xy} < \frac{1}{\sqrt{3}}$$
What is the minimum number of elements of $A$ such that this holds for any set $A$?